Multilevel preconditioning of non-conforming FEM problems

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Preconditioners based on various multilevel extensions of two-level finite element methods (FEM) lead to iterative methods which often have an optimal order computational complexity with respect to the number of degrees of freedom of the system. Such methods were first presented by Axelsson and Vassilevski [O. Axelsson, P. Vassilevski $89\&90$], and are based on (recursive) two-level splittings of the finite element space. The key role in the derivation of optimal convergence rate estimates plays the constant γ in the so-called Cauchy-Bunyakowski-Schwarz (CBS) inequality, associated with the angle between the two subspaces of the splitting. More precisely, the value of the upper bound for $\gamma \in (0, 1)$ is a part of the construction of various multilevel extensions of the related two-level methods.

In this paper algebraic two-level and multilevel preconditioning algorithms for second order elliptic boundary value problems are constructed, where the discretization is done using Rannacher-Turek non-conforming rotated bilinear finite elements on quadrilaterals. An important point to make is that in this case the finite element spaces corresponding to two successive levels of mesh refinement are not nested in general. To handle this, a proper two-level basis is required to enable us to fit the general framework for the construction of two-level preconditioners for conforming finite elements and to generalize the method to the multilevel case.

The proposed variants of hierarchical two-level basis are first introduced in a rather general setting. Then, the involved parameters are studied and optimized. The major contribution of the paper is the derived estimates of the constant γ in the strengthened CBS inequality which is shown to allow the efficient multilevel extension of the related two-level preconditioners. Representative numerical tests well illustrate the optimal complexity of the resulting iterative solver.